Junior High School

Curriculum Guide

MATHEMATICS

(Tentative Edition)

PROVINCE OF ALBERTA
DEPARTMENT OF EDUCATION
September 1952

ACKNOWLEDGMENT

This curriculum guide has been prepared by the Subcommittee on Junior High School Mathematics, under the guidance of the Junior High School Curriculum Committee.

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INTRODUCTION

THE PLACE OF MATHEMATICS IN THE JUNIOR HIGH SCHOOL

General Observations

Until comparatively recent times the inclusion of many subjects in the school curriculum was supported by the doctrine of formal discipline. This doctrine stated that the mind consisted of distinct faculties such as reason, will and memory, and that certain subjects were particularly valuable in training one or the other of these faculties. Mathematics was claimed to be the subject *par excellence* for training the reasoning power.

The doctrine of formal discipline depended for its credibility upon the theory of automatic transfer of training. This implied that the ability to reason learned in mathematics would transfer automatically to any and all other situations in which this ability was required. Subsequent findings in the field of psychology have proved formal discipline to be a false doctrine and have caused the theory of transfer to be modified to a great extent. While there is presently no general agreement as to the exact conditions conducive to the optimum amount of transfer of training, it is believed that the amount of transfer can be greatly increased if teaching methods specifically provide for opportunities to apply mathematical learnings in many and varied situations.

General Objectives in Education

According to the Curriculum Guide for Alberta Secondary Schools, mathematics makes its major contribution to the general objectives in the field of personal development, and more specifically in the area of intellectual achievement. The Curriculum Guide states that intellectual achievement contributes toward personal development and that two important factors in intellectual achievement are "a broad understanding of the fundamental principles of mathematics and their importance in daily living" and, "a mastery of mathematical skills necessary for vocational competence."* While mathematics makes its major contribution in the field of personal development, it makes significant contributions toward all four general objectives of secondary education, in the following ways:

* Curriculum Guide for Alberta Secondary Schools (1950 edition) page 11.

1. *Personal Development*

A knowledge of the mathematics involved in handling funds contributes toward personal development by giving the individual an increased feeling of security in his financial affairs. In addition the field of mathematics presents a system of exact relationships which are logically consistent and aesthetically pleasing.

2. *Growth in Family Living*

Mathematical skills and thinking are required in many activities which enhance family living, such as budgeting income, keeping accounts, buying on time, estimating depreciation and providing security for family members and property.

3. *Growth Toward Competence in Citizenship*

In order to understand more fully the significance of civic affairs the individual needs to have a practical knowledge of the mathematics involved in the administration of various taxes, the use of public funds, and the meaning of statistics dealing with such matters as the distribution of wealth, industrial activity, crop prices, price indexes and interest rates. In addition much general information dealing with scientific and technical advances has meaning to the individual only if he has had some of the appropriate mathematical training.

4. *Occupational Preparation*

It is hardly necessary to point out the need for a knowledge and understanding of mathematics in the world of business. Most businesses require at least one of the following mathematical skills: accurate and rapid calculation, accurate estimating, application of formula, solving problems involving quantitative thinking, and interpreting graphs, charts, diagrams, and tables of figures. Further, mathematics is basic to success in certain professional training. The junior high school prepares the student for a study of the more involved, complex and abstract mathematics necessary for competence in the fields of science, engineering and commerce.

Immediate Value of Mathematics to the Junior High School Student

The student often has immediate need for, and can make use of skills learned in mathematics both in other school subjects and in his out-of-school activities. In subjects such as General Shop, Home Economics and Science the need for and the use of mathematical skills is apparent. In Social Studies it is often necessary to interpret statistical data and to make comparisons involving quantitative thinking. In his daily living the student has many opportunities to use mathematics in activities such as buying and selling in his own interests.

CHAPTER ONE

BASIC PRINCIPLES OF ORGANIZATION

Objectives in Junior High School Mathematics

The objectives for mathematics may be stated as follows:

1. To develop the ability to think clearly;
2. To develop the ability to use information, concepts and general principles;
3. To develop the ability to use fundamental skills;
4. To develop desirable attitudes, especially respect for knowledge, respect for good workmanship, respect for understandings, social mindedness, open mindedness;
5. To develop a wide range of interest and appreciation.*

* Adapted from *Fifteenth Yearbook of the National Council of Teachers of Mathematics.*

(1) The ability to think clearly: This can be developed by the following types of mathematical work:

1. Gathering and organizing data;
2. Representing data;
3. Drawing conclusions;
4. Establishing and judging claims of proof — the present trend toward including informal geometry in the junior high school is an attempt to use geometry to create a critical attitude of mind.

(2) The ability to use information, concepts, and general principles: The function concept, which has so many applications, can be developed in many mathematical situations. It is obvious that there are abundant opportunities to use information and general principles in mathematical work.

(3) The ability to use fundamental skills: The computational part of mathematics constitutes one of the most important skills required in modern life.

(4) Desirable attitudes: These are developed insofar as mathematical work engenders a respect for knowledge and good workmanship in the field of mathematics and other fields.

(5) Interest and appreciation: Mathematical work can lead to an increased appreciation of the significance of the work done by physicists, engineers, astronomers, and other specialists.

Major Areas of Mathematical Study (Scope of the Courses)

The five major areas which comprise the scope of mathematics are as follows:

1. Number and Measurement;
2. Principles of Geometry;
3. Graphic Representation;
4. Algebraic Representation;
5. Functional Relationships.

The achievements to be realized in the junior high school in each of these major areas of study are set forth as follows:

(1) *Number and Measurement*

Naming and recognizing concepts; explaining and defining significant terms; using the four fundamental operations with integers and decimal fractions; using the units of measure; reading and interpreting correctly simple numerical tables such as are found in social studies texts; judging the appropriate degree of accuracy consistent with the data of any given problem.

(2) *Principles of Geometry*

Recognizing, naming and sketching figures; developing formal definitions for whole figures and for parts of figures (e.g. radius, transversal); developing the ability to use common geometric instruments; making direct measurements of lines and angles; using formulas to find the area of common plane figures; making simple indirect measurements; developing the ability to apply such propositions as the angle-sum of a triangle and the Pythagorean relation; the ability to recognize the effects of changes in dimensions (e.g. the change in the area of a square the side of which is doubled).

(3) *Graphic Representation*

Naming and defining such terms as unit, scale, axis; constructing a suitable graph from given data (e.g. circle graph, bar graph); recognizing and reading actual values on a graph; making simple interpolations.

(4) *Algebraic Representation*

Explaining and defining basic terms (e.g. directed number, fraction, raising to a power, coefficient, exponent, equation, evaluation); performing the fundamental operations with general numbers; using the laws of exponents; solving equations; writing the equations implied in verbal problems.

(5) *Functional Relationships*

Recognizing, naming and defining such terms as constant, variable, formula, table; reading tables of related values; evaluating formulas; interpolating from tables and graphs; constructing formulas from verbal statements; selecting the variables in a given problem.

NOTE: The level of achievement for each grade is given in the Scope and Sequence Chart which follows, and in the Course Outlines.

Purpose of Charts I, II and III

Chart I is designed to show the gradual development of each of the five major areas of study in the three grades of the junior high school. Chart II is designed to show what techniques are employed in acquiring a mastery of the computations and other skills. Chart III is designed to show the essential steps in the logical solution of a problem.

Explanation of Chart I

The five essential concepts are expanded as the student moves from grade to grade, and as the nature of the subject demands that its application be broadened. Each concept is the result of slow growth. Three of these concepts have been fairly well defined in the upper elementary grades. The functional application requires very careful integration as the course progresses.

CHART I: SCOPE AND SEQUENCE OF ESSENTIAL CONTENT IN JUNIOR HIGH SCHOOL MATHEMATICS

SCOPE: Major Areas	SEQUENCE (Progressive development of the areas of study in mathematics through the grades)		
	GRADE VII	GRADE VIII	GRADE IX
NUMBER AND MEASUREMENT	The place value of numbers Rounding off numbers Understanding large numbers Common units of measurement Fractional parts	The place value of number extended Approximations Powers of numbers Square root Metric units of length Area and volume Metric equivalents	Signed number Absolute value of number Powers and roots Angular and circular measurement Scales
PRINCIPLES OF GEOMETRY	Straight lines; plane figures; parallel lines; bisectors.	Perpendiculars; congruency; symmetry	Construction; similarity; parallelism; right triangle
GRAPHIC REPRESENTATION*	Pictographs; bar, broken line, curved line	Pictographs; bar, broken line, curved line; circle; interpolation	Pictographs; bar, circle, line, broken line, smooth curve; formulas
ALGEBRAIC REPRESENTATION	Literal representation	Equations; roots of equations; parentheses	Literal and numerical coefficients; monomials; binomials; equations
FUNCTIONAL RELATIONSHIPS	Meaning; methods of indicating ratio	Relationship of number, units, amounts; problems	Problems; lines; angles; areas; units; constant; variable

* The subject matter in these grades requires that the same types of graphs be used, with a gradual increase in the significance of the graph.

Number and Measurement — This is a logical development of the study of number and number values begun in the elementary grades. A clearer concept of the place value, whole or fractional, of number will be established. Methods of measurement, units of measurement and related units form an important part of the pattern of essential concepts.

Principles of Geometry — Certain principles of geometry are established in these grades. The foundation is laid for more extensive study at higher levels. Utmost care must be taken to assure a thorough understanding of the vocabulary of geometry. The whole field of mathematics in these grades may be fortified by clearly defined concepts of plane figures, lines and angles.

Graphic Representation — This offers to the student a new method of expressing certain relationships. While the mechanics of constructing simple graphs have been learned in elementary grades the concept has still to be developed. The pictograph is a good starting point. Accuracy of detail must be stressed from the beginning. (Suggestion: Stick-men of different heights, varying from one-half inch to two inches, may be used to show differences in population of the towns of Alberta, where differences are not too great.) Some of the concepts of measurement and geometry become meaningful when applied to the line and bar graph and the broken line and circular graph. Various means are used to illustrate the total picture of a representative concept. The same type of graph is used throughout the grades of junior high schools with a gradual increase in its significance.

Algebraic representation — A very important step is taken into the field of algebraic representation. Letters are used to represent number, units, quantity and other numerical relationships. Example: Two lengths of string are 10 inches and 12 inches. The length of one may be represented by "a" and the other by "b". When the two are tied together their combined lengths may be represented by (a + b) inches. Written as an equation:

$$(a+b)=22 \text{ inches}$$
$$\text{or } a=(22-b) \text{ inches}$$
$$\text{or } b=(22-a) \text{ inches}$$

Expressions of equality are used as simple equations in which a letter represents a number so that a balance is maintained. The emphasis is upon the value of the parenthetical unit expressed as one quantity regardless of the value given the literal factor. Thus, $2(a-b-c)$ is a mathematical unit with 2 as a co-factor with $(a-b-c)$, 2 being the numerical factor and $(a-b-c)$ the literal factor. Or, more simply in 2ab, 2 is the numerical factor or coefficient of ab, and ab the literal factor. Also 2a is the coefficient of b. All three are co-factors of the product.

Functional Relationships — This area of the scope is closely related to each one of the preceding areas. The relation of number gives us the common fraction, the decimal fraction, the percentage fraction, the ratio of the side of a square to the perimeter and to its area and the constant relationship between the diameter of a circle and its circumference. In problem analysis, the function of the units involved sets up the relationship to be discovered. When the relationship between quantity and cost (for example) is known, other costs and other quantities may be determined. In the simple formula, $C=2\pi r$ the value of C depends upon the value of r. The factors 2 and π in the equation have a constant relationship to both C and r.

Explanation: $C=2\pi r$

$$C=2\times\frac{22}{7}\times r$$

$$\therefore C=\frac{44r}{7}$$

$$\therefore \frac{44}{7}=\frac{C}{r}$$

$$44:7 :: C:r$$

$$\therefore 44r=7C$$

Read Section 2, Comprehension, and Section 4, Interpretation, on Chart III.

CHART II: TECHNIQUES TO BE FOLLOWED IN ORDER TO ACHIEVE COMPETENCY IN COMPUTATIONS AND SKILLS

OPERATION	DETAILS OF OPERATION
Diagnostic Survey	Class survey conducted at regular intervals to determine difficulties in basic operations. Written and oral testing. Record charts to show progress. Extend survey to Grade VIII and IX covering fractions, decimals, and percentages, place values of numbers, whole numbers and integers. A continuous class inventory must be set up for more effective remedial measures.
Remedial Measures	Examine difficulties discovered in diagnostic surveys; select small groups with common difficulties; use controlled procedures (i.e. a close co-ordination between the teacher and class working out the difficulty — a clinical process). Endeavour to establish the habit of correctness at every step. Estimating the result has a definite value in the correction of weaknesses. (Chap. 7 Grade IX).
Accuracy and Speed Improvement	Accuracy first, then moderate speed; stress accuracy in every operation; consistency in reaching a correct result will become a habit; finding simple factors, multiples, powers and roots at sight help in attaining skill in accuracy; a moderate rate of speed, consistent with accuracy, must be a definite objective in junior high school mathematics. (Read Preamble in Detailed Outline for Grade VIII.)
Relationships	"Mathematics . . . the science of relationships." Examine the relation of numbers to unity; compare the value of the number to its tenth and hundredth part; also to its multiples; the relationship between a whole number and its fractional parts; between a fraction and its decimal equivalent; between a fraction, its decimal equivalent and its percentage equivalent; relationship expressed as an equation. Related values (e.g. 2x, 2x—1, 2x—2).
Estimation and Verification	Estimating results; setting up the range of probability; estimating values, areas and costs; class estimations arouse interest. Consider all the related data before the estimation is stated. Estimating budgetary receipts and expenditures. Verification of results strengthens the concept of accuracy. The responsibility for accuracy is an individual matter. Repeat the operation; use the reverse method; diagram method may prove helpful. Verify result by substitution. Verification by applying the angle-sum rule in the study of triangles.

Explanation of Chart II

The five phases of this chart point out in detail the plan of operations which is intended to improve methods in calculation and to develop consistent accuracy. This is a student-teacher chart with the responsibility about evenly divided between the teacher and the student. The student must be encouraged to improve his ability to make correct calculations. The habit of correctness is somewhat more than a suggestion. The teacher discovers individual or group difficulties and plans remedial measures. Teachers will find a gratifying response to a well-organized plan for the improvement of the individual student.

Diagnostic Survey — A thorough survey of all classes at the beginning of the school year is strongly urged. Skill in performing basic operations should show a marked improvement and accuracy, and speed should increase throughout the fall term. However, it will likely be found that individual students have difficulty with one or more of the basic operations. The need for diagnostic testing is recognized in the textbook and test material is found throughout. The key tests on pages 291 to 308 for Grade VII, and on pages 295 to 308 for Grade VIII should prove helpful if discreetly used. Individual record charts should be kept by the student, showing the type of difficulty most often encountered.

Remedial measures — With the recognition of difficulties in computation and other skill techniques we face the problem of correction. Difficulties may be caused by either (a) a vague understanding of the process, or (b) a lack of appreciation of the value of number. The latter may date back several years.

Oral and written drill are useful in correcting faults in the basic operations. The application should be varied. Horizontal as well as vertical addition should be encouraged. The two-way table shown below will illustrate.

Add horizontally

Add vertically

5	8	1	4	5	2	7	4	9	45
2	3	8	9	4	5	6	7	6	50
7	4	5	6	3	4	5	2	3	39
4	7	2	5	8	7	4	5	8	50
3	6	9	8	7	6	3	8	7	57
8	5	4	7	6	9	2	3	1	45
29	33	29	39	33	33	27	29	34	286

The above remedial exercise is made of even and odd numbers placed alternately both horizontally and vertically. Practise adding left to right; check by reversing direction. Add up; check by adding down. The same number should not appear adjacently in either direction. This technique may be effectively used in Grade IX for addition of monomials, signed numbers or signed monomials. Students check their own answers.

Accuracy and Speed Improvement — Various methods are needed in building up a reasonable fluency in the manipulation of numbers. The small group working under a controlled procedure seems to be the most practical method. Frequently a few Grade IX students have difficulty in finding a common denominator for fractions to be added or subtracted. A few simple examples at the board may help to clarify the technique. When the denominators are prime numbers, the common denominator will be their product. In converting a mixed number like $3\frac{1}{8}$ to an improper fraction, the value does not change; it is only written in a different form. It takes twenty-four eighths to equal 3, and one-eighth more will make the converted fraction 25/8. The teacher's objective should be to develop an easy confidence in manipulations of numbers at moderate speed.

Relationships — Mathematics is often referred to as the science of relationships. The teacher's objective in this area is to establish a keen appreciation of the value of number. Since all numbers are related to unity there is a definite relationship between all numerical values. All fractions, including common fractions, decimals and percentages, are parts of the unit and therefore also have a definite relationship with unity. The common fraction shows a relation between a dividend and its divisor. When the fraction $\frac{1}{4}$ is changed to a percentage fraction only the form is changed, the numerical value remains unchanged. The same relationship is maintained. The equation establishes a balanced relationship between quantities or values. Some new relationships are found in geometry. The right angle triangle gives us a whole set of new and interesting relationships extending far beyond the junior high school.

Estimation and Verification — Estimating a result in advance of the basic operation has a definite place at this level. It should provide training in making reliable estimates from the data provided. It should also teach the student that estimates are only guesses which must be corrected by actually performing the operations necessary to the solution of the problem. Estimating the answer to a mathematical problem also has the advantage of drawing the student's attention to the necessity of arriving at a reasonable answer. Estimating a reasonable answer will prevent the student from being satisfied with answers to problems, which result in airplanes travelling at two million miles an hour and gasoline storage tanks having a capacity of billions of gallons. When approximations only are required it is sometimes best to leave a reasonable answer as a good answer. Sometimes the mean of the guesses of all the members of the class may be taken as a basis for calculation. The student should be made aware of the effect of certain mathematical operations such as the fact that when a whole number is divided by a fraction the result will be greater than the whole number. "More" or "less" should never be regarded as estimations unless qualified by "how much." However, the statement "more than twenty dollars but less than thirty dollars" provides definite limits and may be considered as an estimation. The student should be encouraged to consider all the available data bearing upon the problem

before making an estimate. If they do so their estimates will be more reliable.

Certain types of exercises may be profitably checked by the students. Others require checking by the teacher. A good opportunity for limited reteaching is provided where the teacher may select a few weak students for special treatment. Responsibility for accurate checking must be assumed by the students. No teacher has sufficient time to check the entire daily work of his class. Sometimes it is best to repeat the operation in an effort to achieve accuracy. Reverse operations should be carefully controlled. The additive method is an interesting way of checking simple subtraction. The diagram method is widely used in checking answers in terms of area, profit and loss, and discount.

Verifying the result of the solution of an equation by substitution is both direct and conclusive. The angle-sum rule is a useful check in the construction of triangles, quadrilaterals, parallel lines, and the angular points of equilateral polygons.

CHART III: SEQUENCE IN THE LOGICAL SOLUTION OF A PROBLEM

SEQUENCE	DETAILS OF SEQUENCE
Vocabulary	Study words as they become necessary in understanding the problem. Meaning and spelling. Functional meaning in mathematics. Stress the use of proper mathematical terms. Special use of certain words in mathematics.
Comprehension	Accurate reading of problem; selection of "Key Units"; setting up the known relationships. Controlled discussion to establish complete understanding of problem-situation. This step is very important in the logical solution. Isolation of the main idea.
Problem-situation	Recognition of problem-situation; related factors in the situation; establish the relation of known factors to unkown factors; an estimation of the value of unknown factor fits into the sequence. A good sketch or diagram helps to clarify thinking.
Interpretation	All the relevant ideas are translated to the major problem. A clear perspective of what is known and what is required is vital at this stage. The student's ability to construct a clear picture of the situation is considered of greater value than the actual solution. (Read reference to Interpretation in detailed analysis of problems to follow Chart III.)
Solution	Arithmetical solution is a logical step where the interpretation is clear. Problems are the devices used to develop logical thinking. The basic operations must be accurate and complete. Refer again to diagrammatic representation to complete the solution. State the result in words.
Generalization	Express the result as a formula. The formalized conclusion has a definite value. Synthesized problems may be useful in the test of interpretation. NOTE: Avoid the use of expressions like this: A=154 sq. in. or, r=7 in. Use A is 154 sq. in. or r is 7 in.

Explanation of Chart III

Vocabulary — A list of words frequently used is included in the program for each grade. Most of these words are new and become part of the vocabulary of the grade. The vocabulary of mathematics is clear, concise and functional. It is this functional vocabulary which we desire to add to existing vocabularies used in speaking, listening, reading and writing. Treat new words objectively. The word "equilateral" takes its meaning in mathematics from certain plane figures where its application is pertinent. The search for roots and derivatives need not go too far. A clear knowledge of the meaning of words in a problem is necessary for intelligent understanding of the problem-situation.

Comprehension — This involves careful reading to gain the main idea in the problem-situation. A second reading, with or without help, will identify the units to be used. The "key words" set up the framework of the problem, and accurate reading helps in their selection. The "key units" give the problem its stature and provide the known relationships. A thorough comprehension of the problem-situation by the group must be established before further analysis is attempted.

Problem Situation — With a good understanding of the related factors in the problem the class is ready to make use of them in determining the unknown factor or factors. This is a good point at which to make a sketch or diagram to emphasize the dominant factors in the situation. Thus a control is set up on the procedures to follow.

Teacher: Read the analysis of type problems to follow Chart III.

Interpretation — The translation of relevant ideas to the major problem is a logical step. The good judgment of the student is challenged at this stage and the critical point in the analysis has been reached. If the students' perspective is clear, no difficulties will arise. If they do, the whole problem should be given a careful review. Each step in the logical development must be integrated into the system of ideas.

Teacher: Read the analysis of type problems to follow Chart III.

Solution — Working from the diagrammatic representation of major ideas of the problem, the basic operations to be used in the solution should become clear. There is no room here for a trial and error method. Any difficulty in making a clear-cut arithmetical solution would indicate a lack of comprehension or faulty interpretation of the problem. At some point the problem analysis has broken down. The proper procedure in such cases is to review the entire problem area with emphasis on interpretation. It is generally agreed that a few problems which tend to develop logical thinking are preferred to many problems where the only goal is a "right answer." The background of the entire course in mathematics in the junior high school is thoroughness. Therefore, the analysis of the problem is more important than the necessary mechanical operations required in the solution.

Generalization — New vocabulary may be put to good use in the generalization of results. The development of a formula is a logical step after the factors involved have been expressed in words. The formula should not be used until the need for it arises. The area of the rectangle expressed in words

gives meaning to the operation. In the formula, $A=L\times W$, it must be understood that L represents the number of units in the length of the rectangle and W represents the number of units in the width of the rectangle, and since the area A must be expressed in square units, the units in L and W must be the same kind of units. In the circumference formula, $C=2\pi r$, C may be found when r is given; r may be found when C is given, 2π having a constant value.

Examples of the Logical Solution of a Problem

These problems of various types are offered as suggestions. With each problem type is given a detailed analysis of the procedures involved in its solution. Reference will be made to the plan of problem analysis as indicated in Chart III.

Example I

A new tractor outfit can plow a field in six days. An older tractor outfit can plow the same field in eight days. In how many days can the two outfits working together plow the field?

Analysis: The vocabulary of the problem is simple and clear. Accurate reading will show that one tractor does the work in less time than the other. The key units are: "six days" and "eight days", the key words are "working together" (Comprehension). Controlled procedure (questioning) will reveal that one tractor works faster than the other, whether they work separately or work together. (Comprehension) The dominant factor in the situation is one day's work. (Problem situation) This is a recognition of known factors contained in the key units.

Controlled procedure (teacher and class doing each step together) will make clear that the new tractor by itself does 1/6 of the work in one day while the other does 1/8 of the work in one day. (Interpretation) This is the point at which to introduce the diagram, as illustrated below:

One
day's
work

1/6	1/6	1/6	1/6	1/6	1/6

6 days' work

One
day's
work

1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8

8 days' work

The diagram shows the part of the entire field plowed by each tractor in one day. (Translation of ideas.)
(Interpretation)

Now, the part of the field done in one day with both tractors working together will be (1/6 of field + 1/8 of field). By adding these parts,

$(1/6 + 1/8) = 7/24$, we have part of whole field done in one day. (Application of ideas to situation.) Now for the logical conclusion: Will it take 2 days? 3 days? or 4 days? to complete the job. Students sketch a diagram and mark off in spaces of approximately 1/3 (7/24).

Or $1 \div 7/24 = 1 \times \frac{24}{7} = 3\ 3/7$

$\therefore$ The two tractors will take 3 3/7 days to plow the field. (Generalization): Required time is not an average of different rates of working.

Example 2

A rectangle is three times as long as it is wide. The perimeter is 64 inches. What are the dimensions?

Analysis: The vocabulary of this problem may require some review. The words "rectangle", "perimeter" and "dimensions" should be made clear for accurate reading. The important key units may be set down:

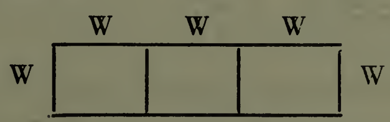
(a) rectangle
(b) three times
(c) perimeter
(d) dimensions

The rectangle is three times as long as it is wide. (Known relationship). The perimeter is 64 inches. (Key word). Applying the concept of literal representation to discover the unknown dimensions, we may represent the width of the rectangle by W. (Translation of ideas). Then the length of the rectangle will be represented by 3W. (Application of ideas to situation).

Students may make a diagram to interpret the major ideas. (Application of ideas to situation)

The perimeter is 64 inches.

Now, "the logical arrangement of ideas."



$\therefore$ (W + W + W) + (W + W + W) + W + W = 64 inches.
$\therefore$ 8W = 64 inches

$W = \frac{64}{8}$ inches, or W = 8 inches

$\therefore$ The width of the rectangle is 8 in.
$\therefore$ The length of the rectangle is (3×8) inches = 24 inches.
$\therefore$ 2L + 2W = Perimeter
2 (L + W) = Perimeter. (Generalization)

Example 3

To construct an equilateral polygon of five sides, inscribed in a circle the radius of which is one inch.

Analysis: Some time should be given to a review of the new terms used in the problem. The key words of the problem may be set down mainly to put the data in an orderly form. (Comprehension)

(a) equilateral polygon

(b) pentagon

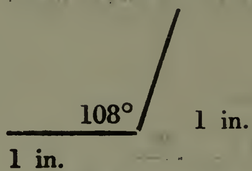
(c) inscribed

The key unit is "one inch." Construct a circle with a radius of one inch. Draw a radius from centre to the circumference, to the right. There are four right angles at the centre of the circle (known factor), or 360°. Controlled procedure will determine the size of each angle at the centre. (Comprehension) Data: $360° \div 5 = 72°$ (Translation of ideas). By using the radius already drawn as one arm make an angle of 72° at the centre. The second arm of the angle will also be a radius. (Application of ideas to the situation.)

Working to the left around the circle, construct five such angles at the centre, arms of the angle being radii, and vertices of the angles being the centre of the circle. (Interpretation) (System of related ideas).

By joining in order the points where the radii meet the circumference, five equal sides are obtained and the inscribed figure is complete. (Logical order in steps of solution). An examination of the figure will show that the pentagon is made up of five equal triangles, all isosceles. By applying the angle-sum rule (checking, Chart II) we find each angle at the base of each of the triangles will be 54°, or 108° at each angular point of the pentagon. (Generalization).

Synthesized problem situation: Construct an equilateral pentagon on a base of one inch.



From this construction the following formula may be derived:

Number of degrees at angular points of any equilateral

$$\text{polygon} = \frac{(N \times 180°) - 360°}{N}$$

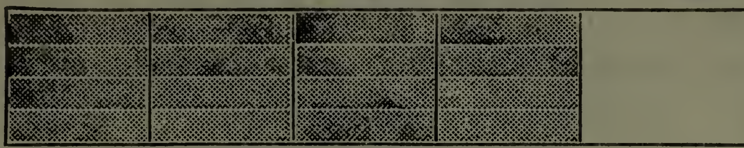
where N is the number of sides of the polygon.

Example 4

When the cost of a dozen eggs increases from 36 cents to 45 cents, what is the per cent increase?

Analysis: The problem situation contains only two key units, "36 cents" and "45 cents." These two units set up the problem where an increase occurs. (Related units). Controlled procedure shows the increase to be 9 cents. (Comprehension). Using 36 cents as the base, and the amount of increase

as 9 cents we have the dominant factors in the situation which may well be illustrated:



(diagrammatic representation)

We can now show the increase as a part of the base. (Translation of ideas). Thus the increase is 9/36 of base. Or better still, 1/4 of base. (Application of ideas to the situation).

Then 9/36 expressed as 100ths is.

$9/36 \times 100 = 25$

$\therefore$ The increase in cost is 25%.

Synthesized problems based on Example 4.

(a) Express 36 as a percent of 45.

Referring to the diagram, it is seen that 45 is made up of 5 equal sections, and 36 is made up of 4 equal sections. Then the part which 36 makes up of 45 is written as 36/45 or simply 4/5. Then, 36 is 4/5 of 45. (Generalization).

$\therefore$ $\frac{36}{45}$ expressed as a percentage

$4/5 \times 100 = 80$

Then, 36 is 80% of 45.

(b) Express 45 as a percent of 36.

Referring to the diagram in Example 4 we find that 45 is made up of 5 equal sections, while 36 is made up of 4 sections. (Picture of the problem).

More simply, 36 equals 4/5 of entire figure, and 45 equals 5/5 of entire figure. (Translation of ideas)

Then $\frac{5/5\ (45)}{4/5\ (36)} = \frac{5}{5} \times \frac{5}{4} \qquad \left(\frac{45}{1} \times \frac{1}{36}\right) = \frac{5}{4} \qquad \frac{5}{4} \times 100 = 125$

Therefore 45 is 125% of 36.

Example 5

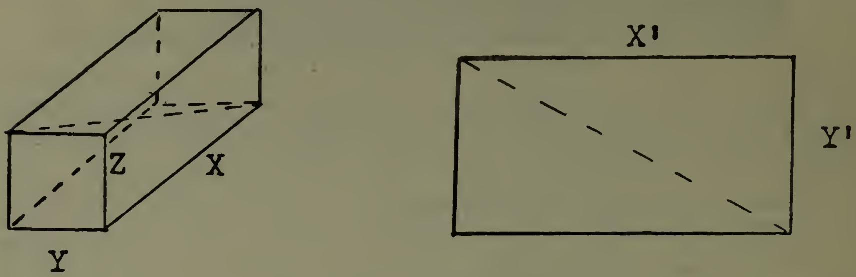
A box is x feet long, y feet wide and z feet in depth. Write an algebraic expression for the length of the shortest distance from one upper corner to the farthest corner at the bottom of the box.

Analysis: The vocabulary used is quite familiar to Grade IX students. The relationship of units is readily seen where an open box is used. to illustrate the given dimensions.

A knowledge of the Pythagorean theorem is a prerequisite in this solution. (System of related ideas). The bottom of the box is a rectangle. x feet long

and y feet wide. (Dominant factors). The diagonal of this rectangle will be the length of a straight line from one corner to the opposite corner on the bottom of the box. (Interpretation).

This line will be the hypotenuse of a right angle triangle, and its length will be represented by $\sqrt{X^2 + Y^2}$. (Application of ideas to situation).



(diagrammatic representation)

Now, a second right angle triangle requires solution. Its base is represented by $\sqrt{X^2 + Y^2}$ and its altitude by Z. (System of related ideas). There is a good exercise in spatial thinking here that should not be overlooked.

We visualize an "air line" as the hypotenuse of the second triangle.

Since Z is the height of a right angle triangle the base of which is $\sqrt{X^2 + Y^2}$

Then the square on the hypotenuse $= \sqrt{X^2 + Y^2}^{\,2} + Z^2$

Or, the required hypotenuse $= \sqrt{X^2 + Y^2 + Z^2}$.

Use this formula in all problems of this type.

CHAPTER TWO

THE GRADE SEVEN COURSE

Field Chart showing the relation of major areas of study in Grade VII Mathematics to the authorized text.

MAJOR AREAS (Scope)	References to Mathematics We Use, Book I.
I. Number and Measurement	Chapter I, II, III, V, VIII, IX. "Let's Practice" tests throughout the text.
II. Principles of Geometry	Chapter VI and VII.
III. Graphic Representation	Chapter IV; elsewhere throughout the text; "Let's Practice" scores throughout the text.
IV. Algebraic Representation	Perimeter and area formulas; Chapter VII; Interest formula pages 265 and 266.
V. Functional Relationships	"True and False" tests, p. 57; Problems without numbers, p. 18 and 112; Batting and Scoring ratios; Scale drawings, p. 159 and 162; Length - Perimeter - Area relationships, Chapter VII; Interdependence of cost and quantity; speed, distance and time; other inter-relationships shown in graphs and problems throughout text.

OUTLINE OF THE COURSE FOR GRADE VII

Text: *Mathematics We Use, Book I.*

References for Remedial and Enrichment Materials:

Mathematics and Life, Book I.

Everyday Junior Mathematics, Book I.

Preamble:

Since the course in mathematics for the first year of the junior high school represents more or less a transitional step from the arithmetic of the elementary grades to that of the wider field of learning in mathematics, the teacher of Grade VII would be well advised to keep in mind the point of view as expressed at the elementary level, viz.:

(a) Simply assigning pages to be read and exercises to be done and corrected cannot be considered satisfactory mathematics teaching.

(b) Meaning must be established before memorization is required.

(c) Qualitative understanding involves special reading skills and study skills.

(d) Every junior high school teacher must accordingly maintain a reading program in mathematics to match the progress of his class. In the new text for Grade VII the following features may be noted:

1. The basis of instruction throughout is (i) the number system and (ii) its social significance.
2. There is a scientific graduation of subject matter.
3. Teaching procedures throughout insure mastery and understanding of mathematics. One in particular that cannot be emphasized too strongly, is the use of diagnostic tests and remedial practice exercises. These make it possible for the teacher to provide effectively for individual differences.

(e) Approximations must be used in nearly every teaching lesson, and proper steps in solving problems must be taught. (See Logical Thinking in Problem Analysis following Chart III.)

Detailed Program — Grade VII

1. *Review of Fundamental Operations*

It is the teacher's obligation to analyze the pupil's performance and to discover strong and weak features. Remedial work must be a follow-up of the diagnosis. Such work requires the keeping of an accurate case-record for each pupil, and constant reference to this record. The diagnostic testing procedure covering whole numbers, fractions and decimals is outlined on page 6 of the text.

The following tables give a convenient summary of the tests and remedial work as found there. Other diagnostic and remedial work is suggested later in this outline.

DIAGNOSTIC TESTS FOR GRADE VII (page 6)

Process	Page Number for		
	Whole Numbers	Fractions	Decimals
Addition	9	44	67
Subtraction	11	47	68
Multiplication	15	49	73
Division	22	55	90

REMEDIAL WORK FOR ABOVE TESTS

Process	Page Number for		
	Whole Numbers	Fractions	Decimals
Addition	291 - 292	299 - 300	304 - 305
Subtraction	292 - 294	300 - 301	305 - 306
Multiplication	294 - 295	301 - 302	306 - 307
Division	296 - 298	303 - 304	307 - 308

It is recommended that teachers review whole numbers, fractions and decimals separately, with diagnostic tests given as they are reached in the first three chapters so that too much diagnostic and remedial work is not attempted at one time.

2. *Division by a Three-figure Number*

Teachers should not take it for granted that the pupils have mastered division when they reach Grade VII, even though this topic is in the Grade VI text. Scientific grade placement of topics indicates that easy and meaningful learning of this topic is more likely to take place in Grade VII.

3. *Rounding Off Numbers*

Questions on this topic are usually worded, "Round off to . . . " (a certain place name). Therefore it is advisable to work in pages 23 and 65 of the text in conjunction with this topic.

4. *Fractions*

Chapter II of the text combines a review of fractions with a group of exercises based on the topic of communication. It may be found advisable to teach these topics separately. A very interesting introductory lesson on communication can be prepared by using the material on page 39 of the text. Do not let the students use the textbook because in Question 4 a serious mistake is made when the students are told the answer instead of being challenged to work it out. Committee work could be organized profitably on the different kinds of communication.

Although the four mechanical operations in the manipulation of fractions have been reviewed separately in the text and the suggested remedial work carried out, it is often found that errors in these manipulations will still occur when the student is required to demonstrate a mastery of all four operations. It is time well spent to give a diagnostic test to discover processes which have not been learned thoroughly or processes which have been confused; e.g., a student may be found to be reducing fractions to a common denominator before multiplying.

If diagnostic tests are not readily available, a suitable test may be made up. In such a test, addition, subtraction, multiplication and division of fractions should be intermingled. Each should proceed from the easiest to the most difficult. The first example in addition of fractions to occur in the test should be one involving no change in denominator, in the second example of addition to occur, a change of denominator will be necessary in only one of the fractions, in the third, both denominators will have to be changed. Similar gradations in difficulty will occur in the other three processes. The diagnostic tests in the text show the required gradations for each process separately. In all cases, the numbers involved should be simple so that there will be no confusion due to cumbersome manipulations.

The first test of this type given should be marked by the teacher and a record made of the score of each pupil, with the type of error made written opposite the pupil's name. Errors common to a small group should be taken up with only the group involved, with one pupil using the chalk while the others are prepared to be called upon to help out or to offer advice. Errors common to only two or three pupils should be dealt with individually.

If two or three tests are made up, a typical remedial lesson might proceed in this manner: Five minutes spent with the whole class pointing out common errors and errors which require only to be brought to the attention of individuals with no need for elaboration; then three-quarters of the class

proceed with the second test for 20 minutes. In the meantime, five minutes is spent with the small group on a common error, 15 minutes spent giving individual assistance to them in correcting some of the representative mistakes they made in the test. The second and succeeding tests can be marked by the class.

The next lesson might involve a different small group. Pupils keep a record of their scores and note improvement in speed and accuracy. About 30 examples on the test is usually sufficient to keep the better pupils working at top speed.

5. *Decimals*

Place value before and after decimal point; multiplying and dividing by 10 or multiples of 10; the four mechanical operations in decimals; changing common to decimal fractions and vice versa; social value of decimals; problem-solving with decimals. A diagnostic testing should be carried out as in Section 4.

Although decimals have been introduced in Grade VI, the concept is relatively new. The topic should be introduced in problem situations so that there is understanding before manipulation. The advantages of decimal fractions over common fractions should be fully appreciated throughout this unit. This may be illustrated by comparison of games won to games played by teams and by ease of manipulation.

Pupils will quite willingly change common fractions to decimal fractions if they are using them to find out standings in the American or National League, a baseball player's batting average, or a goal-keeper's goal average. (The title "Percent" on National League standing, p. 79 of the text is confusing to pupils.)

Roman numbers might be taught in such a manner as to give an appreciation of the value of the decimal system. The text coverage could stand expansion. By anticipating errors such as XM for 990, ID for 499, teachers might prevent their occurrence.

6. *Graphs*

Bar graphs; line graphs; circle graphs.

Some teachers might wish to complete this topic earlier in the year so that use could be made of graphs for pupils' record work on diagnostic tests, "Let's Practise" exercises and progress tests. Furthermore, as graphical representation is used extensively in many exercises, it would aid greatly in the understanding of other sections of work. Circle graphs should be taken up in connection with a geometry section of the Grade VIII course.

Procedure for the construction of graphs is indicated on page 106 of the text, and graphs are used profusely throughout the entire text. Teachers should encourage pupils to make titles brief and to avoid repetition of information given elsewhere on the graph. For example, on page 104, the title might be "Auto Accident Fatalities" and the number at the left labelled "Deaths per 10,000 Vehicles."

7. *Percentage*

Percent as a fraction with a constant denominator; changing decimals and common fractions to percent and vice versa; finding a percent of a

number; finding what percent one number is of another; social value of percent; problems involving percent.

Changing a fraction to percent by multiplying by 100 is artificial and should not be used. It weakens the concept of percent. See text, page 126.

The topic "Finding a Number When a Percent Of It Is Given" has been omitted from the text. Certain classes might be capable of extending their Grade VII work in this, or it could be used for enrichment material.

8. *Geometry*

Lines — kinds, segments of, parallel lines; angles; how to letter and read angles; parts of a circle; concentric circles; how to construct regular figures in circles; pentagon; hexagon; octagon; correct use of protractor; bisection of straight line segment and of angle by construction.

It is advisable to plan the mathematics lessons to allow for one period of geometry per week instead of waiting until Chapter VI of the text is reached, and then taking it up page by page. Geometry is another major phase of the mathematics program and it is essential that carefully prepared basic ground work should be laid to serve as the foundation for the geometry work which will follow in later grades. Boys who take shop work as an exploratory subject are usually taught the construction of regular geometric figures. Sometimes they might be able to suggest new methods for the class to try. Extreme care should be taken in use of protractor. Students must know the classification of angles and they should be instructed on how to use the number scale on the protractor in both directions. When taking up the bisection of an angle, have the students bisect the three angles of a triangle and produce the bisectors. They will have an automatic check on their bisections if the three bisectors meet at a common point.

9. *Area and Perimeter*

Perimeter of rectangles and circumferences of circles with formulas; areas of rectangles, squares, parallelograms and triangles with formulas. This part could be used as a continuation of the weekly geometry instruction if additional material is needed.

The parallelogram offers an excellent opportunity to establish the concept of functional relationships. The perimeter of a parallelogram is a function of the base and the slant side and is not a function of altitude; area is a function of base and altitude but not a function of the slant side.

Careful teaching can give students an appreciation of precision in definition of terms, and some understanding of the distinction between necessary and sufficient conditions. They should understand that a square is a rectangle but a rectangle is not necessarily a square; a rectangle is a parallelogram but a parallelogram is not necessarily a rectangle.

Algebraic representation is introduced in this unit.

10. *Mathematics in the Home*

How to read gas meters, electric meters, water meters; board feet; budgets. Meter reading should be made very practical to students living in urban centres or in areas where rural electrification has been established. Otherwise no great emphasis need be placed on it.

11. *Mathematics of Business*

Opening a bank account; deposit slip; current and savings account (pamphlets on banking available from local banks); sending money; borrowing money; using the interest formula; commission; trade discount.

Sections 10 and 11 which involve the social applications of number relationships learned throughout the course, should be considered of prime importance and a very practical use be made of them since these are the features which the student of today will need in his everyday life as the adult citizen of tomorrow.

12. *Vocabulary (Meaning and Spelling)*

Pupils are expected to master the meaning and spelling of all new words in the text.

Here are some of the words and phrases, old and new:

abacus
acute
addition
altitude
angle
annexing
approximate
arc
area
average

bank balance
base
bisect
bisector
board foot
British Thermal Unit
budget

cancellations
cancelled cheque
capacity
cashbook
centre
cheque
cheque stub
circle
commission
comparison
concentric
consumer
Credit Union
Current account

date of maturity
day letter
decimal
denominator
deposit
depositor
diameter
discount
dishonoured
dividend
division
divisor

electricity
endorse
estimate
exchange
expenditures
expense
formula
geometry
graph
gross profit
horizontal
improper fractions
income
interest
inventory
invert

kilowatt-hour

label
length
light year
like fractions
linear measure

margin
marked price
model
money order
multiplication
mathematics
measure
mixed fractions
multiplier

net price
night letter
numerator

obtuse
octagon

parallel
parallelogram
passenger-miles
patronage dividends
payee
perimeter
perpendicular
personal
place value
plane figure
practical
practice
practise
principal
product
promissory note
protractor

quotient

radius
ratio
receipt

rectangle
reduce
regular hexagon
remainder
retail
round off numbers
rural

savings account
scale
segment
signature

solid figure
square
straight angle
subtraction
system

telegram
thermometer
trapezoid
triangle

unlike fractions

urban

vertex
vertical
volume

wholesale
width
withdraw

zero

Besides doing vocabulary study on new words as they arise, the mathematics instructor may co-operate with the teacher of English by supplying him with a list of words to be used for spelling exercises.

CHAPTER THREE

THE GRADE EIGHT COURSE

Field Chart showing the relation of major areas of study in Grade VIII Mathematics to authorized text and to other authorized references.

MAJOR AREAS (Scope)	Mathematics We Use Book 2	Mathematics and Life Book 2	Everyday Junior Mathematics, Book 2
I. Number and Measurement	Chap. I; Chap. III; Chap. V; Chap. VI; Chap. XI; part of Chap. XII, pp. 295 to 308.	P. 400-404; Chap. I, II, III. "Learning Through Practice"; "Without Pencils"; Self-testing drills, 1-15; "Self-help", 1-42.	Part of Unit I, Chap. I; Unit II, Chap. IV, V, XIV; Unit IV, Chap. IX, X.
II. Principles of Geometry	Chap. II; Chap. X; part of Chap. XII.	Chap. I, p. 36-56; p. 288 to p. 301; p. 310-343; p. 443-445; p. 127.	Part of Unit I, Chap. II; Unit III, Chap. VI, VIII; Review Unit, Chap. XVI.
III. Graphic Representation	Chap. IV; pp. 63, 64, 145, 153, 176, 185, 186; All "*Let's Practice*" scores throughout text.	P. 162; pp. 178-182; pp. 280-290; p, 69; pp. 162-164; p. 259; pp. 320-321; pp. 290-294; pp. 324-326.	Unit I, Chap. III; p. 115, p. 166; pp. 230-231; pp. 70-85; pp. 252-253.
IV. Algebraic Representation	Chap. XIII; Chap. II; also useful in Chap. II.	pp. 406-409; pp. 410-415; pp. 416-421.	Chap. XI, XII, XIII.
V. Functional Relationships	Chap. V; Chap. VIII; Chap. IX; Throughout entire text.	Chap. III, IV, VI. Refer to Index of Mathematical Content pp. 506-511.	Chap. VII-XIV; All Chapters of Unit IV. Elsewhere throughout the text.

OUTLINE OF THE COURSE FOR GRADE VIII

Text: *Mathematics We Use, Book* 2

References for Remedial and Extension Material:

Mathematics and Life, Book 2

Everyday Junior Mathematics, Book 2

Preamble:

The course in mathematics for the second year of junior high school is contained in the detailed outline. The year's program is divided into 12 general topics. The order in which they appear in this outline is not to be considered as a rigid sequence. The logical sequence may suggest itself as the program develops. Readiness for the next step will become clear as the necessary basic skills are mastered, within the limitations prescribed by individual differences.

With specific reference to the program in foundation geometry, it may be advisable to review the work of Grade VII in Chapter VI, Book 1 of the text, with special reference to triangles, rectangles, parallel lines, parts of a circle, and use of the protractor. Also, words in the Grade VII vocabulary list which are used in geometry should be carefully reviewed. It is suggested that one period a week be devoted to the sections on geometry and algebra.

There is wide agreement for the belief that the main purpose in teaching mathematics at the junior high level is to encourage the student to think for himself. If a clear understanding of the problem situation is achieved and a consciousness of the number relationships involved, the application becomes more or less mechanical. Then, the problem analysis becomes of more value in the development of clear and concise thinking than the solution of the problem.

Diagnostic testing and the remedial follow-up have a place in the program throughout the year. In diagnostic testing the difficulties in operational method are discovered. Remedial teaching requires great skill and care. Here concepts become clear and well-established, or more or less hazy and of little value. Uniform speed must never be considered as important in the correction of operational difficulties. Accuracy first, then moderate speed. A very useful technique is found in taking the whole class back to a point where everyone may succeed in making a perfect score. Thus we encourage the "habit of correctness."

Students should be encouraged to express themselves freely in the language of mathematics. This helps to give a certain clarity to their thinking. The language used should be clear and direct. In every phase of teaching and learning the language factor is of the utmost importance. The ability to express the idea in concise terms indicates a growth toward the major objective in the study of mathematics. When students are able to state why and how a certain operation is performed, they build their power of expression in the use of mathematical terms. It is urged that more time be spent in the directed analysis of the problem situation than in the formalized methods of solution with an answer as the ultimate achievement. Individual thinking may be encouraged by giving the student the opportunity to express himself freely on the relationships of the known quantities in the problem. The one-step problem has definite value in this respect. The relationships are easily understood. Later, when several number relationships are used the idea of a formula follows quite logically. (Example: To find the cost of 10 bushels of grain at $1.45 per bushel; the channel of thinking is quite clear; only one step is needed to arrive at the cost. The thinking process involved is to take the cost of one bushel 10 times). When simple interest is to be found there is a plural relationship to consider. The formula is used as a guide in setting up a balanced relationship between the four factors involved. Here again, the "why" is of greater importance than the mechanical operations necessary.

The use of filmstrips in teaching mathematics is found to be of great value. Wherever these aids are available they should be used as a means of reinforcing the teaching method. It is found that many students are able to understand relationships more clearly when they are presented pictorially. The Plane Geometry Series will be found to be helpful in dealing with lines and angles, surface measure, areas of parallelograms and volume.

The filmstrip may be shown when the topic is presented for the first time; and again at some later time when the topic has been more fully presented.

In general, the program in mathematics will provide for a year's growth in these areas:

1. A reasonable accomplishment in the basic operations.
2. Use of formulas in problem solution.
3. Functional application of percentage.
4. The common fraction and the decimal fraction.
5. Measurement of areas and volumes.
6. Common operations used in business.
7. Simple principles of geometry.
8. Use of algebraic expressions to designate quantities.
9. Continued use of graphic representation.

Detailed Program:

1. *Whole Numbers*

Diagnostic survey in the basic operations. Develop the habit of correctness.

Remedial work whenever difficulties are apparent (p. 295 of the text).

Rhythmic addition, with metronome as a control.

Rounding off numbers.

Place value of numbers.

Frequent oral drill in the basic operations.

2. *Fractions*

Unit fractions, proper fractions, improper fractions, whole number and a fractional part.

Unit fractions (¼ is a way of writing one of four parts into which the whole unit is divided).

Common denominator in addition and subtraction.

Diagnostic survey of method used in multiplication and division.

Frequent drill in converting one type of fraction to another. Converting the common fraction to its decimal equivalent (p. 24 of the text). Percentage fraction, with its constant denominator of 100. Percentages greater than 100 (still with the constant denominator). Percentages less than one per cent. (Sight method — ¼% of $840.00 is read, 1% of $840.00 is $8.40. Then, ¼ of $8.40 is $2.10). Avoid using 1/400. Finding a number from its part expressed as a fraction of the number. (The fraction may be in the form of a percentage.)

3. *Vocabulary*

Review vocabulary list for Grade VII. Spelling and functional use. Family words. (These lists may be mimeographed or hectographed early in the year and studied as the occasion for their use arises.) Provide each

student with a complete copy of the list for this grade. Correlate with work in Language (list included in program).

4. *Denominate Numbers*

Diagnostic survey of the common measures of length, time, distance, area, volume in cubic units, capacity in commonly used measures (pint, quart, gallon, bushel).

Simple addition, subtraction, multiplication and division of denominate quantities.

Metric system units of length, weight. (These units have little value if their study is confined to learning the tables.) The approximate equivalent of the metric measure to common measures of length widens the concept of measure (pages 29 and 294 of the text).

5. *Problem Solving*

One step problems. Two or more step problems, either concurrent or consecutive, solution. Analysis of problems. A clear understanding of the problem-situation must precede the attempted solution. The parts of the problem are related in some way.

Certain relationships are given. Other relationships have to be found by using those already known. It is the method of using these relationships that gives this topic its importance in this grade. (Example: Find the proceeds of a sale amounting to $320.00 when the rate of commission is 5%. The amount of the sale is $320.00, called the base. The commission rate is 5%. Then, the commission will be 5% of the amount of the sale. That is, $16.00. The commission must be paid out of the amount of the sale, that would be $320.00 — $16.00, or $304.00. The selection of the key words in the problem gives ability in recognition of important factors in the situation. Progress tests: Pages 89, 104, 124, 150, 175, 196, 221, 246, 273, 292, 293, may be analyzed before the mechanical operations are attempted.

Business Problems:—Simple interest, instalment buying, compound interest, assessment of property, mill rate, calculation of property tax, discount, successive discounts, costs, profit and loss.

Business Forms:—Receipt, cheque, note (demand and time), deposit slip.

6. *Geometry*

Review the outline for Grade VII in this Curriculum Guide:

(a) Use of protractor. Angles from 1° to 360°.
(b) Bisection of lines and angles.
(c) The circle. Diameter, radius, arc, sector, chord, and concentric circles.
(d) Rectangle, square and parallelogram.
(e) Words frequently used in geometry from Grade VII vocabulary list.

Constructions: (page 205 following of the text)

Bisection of a line segment. (Measure for accuracy.)

Bisection of an angle. (Check the result.)

Draw a perpendicular to given line from a given point on the line.

Construct a line parallel to a given line. Check angles for equality.

Construct a regular polygon with angular points on the circumference of a circle.

NOTE: The study of polygons should deal mainly with those of five, six, and possibly eight sides. Make use of the geometric names for these polygons. List things known to have five, six or eight sides. Example: Hexagonal pencils, octagonal windows, snowflakes.

Special Note on method for problem 6, page 211 of the text: There are 360 degrees at the centre of the circle in which the polygon is to be inscribed. Divide 360 degrees by the number of sides in the required polygon. Since the sides are all the same length, they will subtend equal angles at the centre. The method of "laying off" the points on the circumference may be very inaccurate.

Symmetrical forms. Plant and animal life.

The axis of symmetry:

Point, line and plane as applied to symmetry.

Symmetry as a co-factor of congruency.

Geometric calculations:

Perimeter — Rectangle, triangle, square, circle, polygon.

Area — Rectangle, triangle, square, parallelogram, trapezoid, circle.

Right triangle — Hypotenuse, altitude, base. Calculation of unknown side.

The introduction of algebraic symbols as a form of measurement at this stage arouses a new interest in a rather dull procedure. It also gives meaning to work in formulas taken later.

7. *Graphs and Graphic Representations* (pages 91-102 of the text)

Pictographs: Data may be found in Social Studies, Science and Health.

Bar graph: Horizontal and vertical. Join the "tops" to introduce the broken line graph to be taken later.

Circle graphs: Review pages 176, 177 and 178 in Book 1 of the text series. Data from Social Studies, Science and Health are very useful in illustrating percentages, profit and loss, composition of the air, exports, imports and budgets.

Broken line graphs: Use simple data, individual scores, class medians, temperatures, cost-of-living indexes.

NOTE: It is considered advisable to complete Section Six of the outline before the section on graphs is undertaken.

Standard Time: Graphic representation of the standard time zones. Use mimeograph or hectograph maps of the world and Canada. Indicate local variations.

8. *Ratio and Proportion* (Chapter XII, pages 247-254 of the text)

Ratio may be defined as the quotient resulting from the division of one quantity by another of the same kind. Every common fraction is a ratio. The ratio of one quantity to another quantity is written 3 to 5; or 3:5; or the fraction 3/5.

Simple uses of ratio: Inch to foot (1 to 12, similar units); side to perimeter; cost of one to cost of several; radius to diameter; diameter to circumference; circumference to diameter.

Proportion may be defined as two or more ratios which are equal to each other. 3:5 is equal to 6:10 is equal to 12:20 or, $3/5 = 6/10 = 12/20$. The concept of proportion is well illustrated by scale drawings. Perfect squares and square root (use of inspection, factoring and tables).

9. *Algebra*

Algebraic representation of number. The letters of common formulas are used to represent numbers, known or unknown. Extensive use of algebraic symbols should be made to develop familiarity with this new way of representing numbers, quantities and values. When the quantity A is added to a quantity B, the sum is written, $A + B$.

Use of parentheses to indicate a single quantity.

Adding a known quantity to an unknown quantity.

Taking a known quantity from an unknown quantity.

Multiplying an unknown quantity by a known quantity.

Dividing an unknown quantity by a known quantity.

These operations are essential before any work in equations is undertaken; useful correlations may be made using perimeters, costs, distances, etc.

Equations: Equations may be understood here to be an algebraic statement of a balanced equality. However, the balance is only true if the unknown quantity has a certain value. This value is called the root of the equation.

Building the equation: Using only one unknown.

The Golden Rule of Equations (page 278 of the text):

Solution of equations: Application of the four basic rules to the solution.

Simple equations involving one unknown quantity.

Common formulas using algebraic symbols to express equality.

Simple illustrations of the use of signed number. Use both the vertical and horizontal line to show positive and negative values. The place of zero between positive and negative numbers.

10. *Vocabulary* (Meaning, spelling and use)

Students are expected to learn the meaning and use of new words and phrases in the program in mathematics for this grade. They should be encouraged to use these and other new words freely in discussions. Thus their powers of expression are increased to grade level.

ad valorem duty
amount
angle of elevation
annuity
assessed valuation
axis of symmetry

beneficiary
*bisect
Blue Cross
bond
*budget

cash ticket
coincide
compound interest
congruent
customs duties
customs house

*date of maturity
days of grace
demand note
direct measurement
*discount
*dividend
divine proportion
dockage

elevator capacity
endowment policy
equivalent
excise tax
exemption

face of note
face of policy
face value

graded storage receipt

gross weight

handling charges
hospitalization
hypotenuse rule

income tax
indirect measurement
inscribed polygon
instalment buying
interest formula
interest tables
internal revenue

legs of a right triangle
life expectancy
limited-payment-life policy
line symmetry

make of a note
margin
market price
mill
mortgage

negotiable note
net price
*net weight

*octagon

par value
parallel lines
parenthesis
payee of note
pentagon
per cent
perfect square

*perpendicular
plane symmetry
point symmetry
policy holder
premium
*principal
profit
*promissory note
property damage
property tax
public liability

rate of interest
*ratio
real estate
regular polygon

sales dollar
savings bank
scale drawing
segment of a line
selling price
shrinkage
sinking fund
specific duty
stock certificate
stockholder
storage rates
straight-life policy
successive discounts
symmetry

tariff
tax rate
taxable income
taxes
terminal elevators
time note
trade discount

*The starred words and phrases appear in the list for Grade VII.

CHAPTER FOUR

THE GRADE NINE COURSE

Field Chart showing the relation of major areas of study in Grade IX Mathematics to authorized text.

MAJOR AREAS	Mathematics for Canadians, Book 1
I. Number and Measurement	Chapters I, II, VI, VII, VIII, and IX. Chapters VI, VII and VIII are the most important chapters referring to this area.
II. Principles of Geometry	Chapters I, II and XII are the most important chapters referring to this area.
III. Graphic Representation	Chapter IX.
IV. Algebraic Representation	Chapters III, IV, V and X.
V. Functional Relationships	Found throughout the entire text.

OUTLINE OF THE COURSE FOR GRADE IX

Text: *Mathematics for Canadians, Book* 1

References for Remedial and Enrichment Materials:

Your Mathematics

Mathematics We Use, Book 3

Everyday Junior Mathematics, Book 3

Preamble:

Although the following outline follows the order of the text, the teacher need not feel obliged to conform strictly to this arrangement of presentation in the classrom. It is quite possible that some teachers may prefer to follow the pattern established in Grades VII and VIII, which would mean that instead of beginning at Chapter I the course would open with the review of integers contained in Chapter VI. Many teachers may wish to take the chapter on signed numbers earlier in the course. The necessity of doing this can be avoided if, in the solution of equation, the x's are gathered on the side of the equation containing the greater number of x's. The exercises in spatial imagination and those at the end of each chapter form an integral part of the course. (Of course, not all pupils will finish all the problems.) The questions under "Recreations" are optional and are intended for the better students. Our aim should be to have the pupils develop the faculty of clear thinking, skill in using formulas and solving problems so that they will be able to appreciate the maze of scientific development which stems from an ever-increasing body of mathematics.

Detailed Program, Grade IX:

(The material in the text is considered sufficient when no comment is made.)

The Drawing of Accurate Figures (Chapter 1)

Accurate drawings have value for the following reasons:

1. The pupils will use them in this course to obtain evidence of the truth of statements regarding geometric figures.
2. Problems will be solved by scale drawings.
3. Student activity will take place leading to knowledge of geometric figures.
4. Pupils will realize that a measurement is accurate only within certain limits. This idea will be tied in with the work on rounding off numbers, significant digits, product of approximate numbers (pages 81 to 87) and quotients of approximate numbers (pages 241 and 242).

Scale Drawing: The techniques used in drawing accurate figures referred to in this outline should be followed in all scale drawings. The answers obtained by means of scale drawings are, of course, approximations and the accuracy of the answers depends on the care with which the drawing is made. In the work on scale drawing the aim is accuracy and understanding of the situation.

Some Techniques for Drawing Accurate Figures:

1. Use a sharp pencil in order to obtain fine lines.
2. Use dividers (or compass) in measuring distances.
3. In drawing a line segment of a specified length, draw a fine line, then cross the line by a fine short line in order to designate the starting point (a) of the segment, thus:

(a)

4. Use care in placing the protractor when drawing or measuring an angle.
5. For all but the most elementary drawing make a preliminary free-hand sketch showing: the true length and the scale length of each line; the size of each angle and the letters associated with each point. In this way the student can interpret the problem. If the student stops to draw the figure accurately in the preliminary stage he is likely to lose track of the essential plan.
6. It should always be understood, where the pupil is free to choose the scale to be used, that one should be used which will give the largest figure possible on the work page. An error of 1/16 inch made on a large drawing is less serious than an error of 1/16 inch made on a small drawing. (Understanding of this point is needed again in the work involving the derivation of formula from empirical data; see pages 177 and 178.)

Proof:

While the course does not cover the subject of formal proof, it is desirable that the students understand that they are not proving a statement when making measurements. They are only offering evidence as to the truth of the statement. See "A warning about generalizations" on page 39 of text.

Concepts:

1. Horizontal, vertical, oblique: In addition to an understanding of these terms obtained by examples found in the immediate neighborhood, the pupils should probably see "vertical" as a radius of the earth and "horizontal" as a tangent to the surface of the earth. These latter ideas will be useful in the science course in the work on angle of latitude and altitude of the North Star.

2. Circle: The path traced out by a point moving on a plane surface so that it is always at a fixed distance from a fixed point. (In describing a circle or an arc of a circle the location of the centre and the length of the radius should be given. This should be kept in mind when a student is required to tell how to carry out certain constructions such as: bisecting a line with compass and straight edge.)

3. Polygon, quadrilateral, parallelogram, rectangle, square, rhombus (what they are; the relations between them; how to construct).

4. Angles: An angle is an amount of turning or rotation. To emphasize this it is suggested that the pupils be asked to turn through certain specified angles, such as 90, 270, 180, 360, 450 degrees. The static figure used to represent an angle (two arms and a vertex) shows the initial direction, the final direction and the point about which the rotation takes place.

   Stress the meaning of a complete revolution and make clear the distinction between rotation and revolution. Introduce the concept of the right angle as one-quarter of a complete revolution, the straight angle as one-half of a revolution and a 45-degree angle as one-eighth of a revolution.

   The use of the protractor for measuring an angle and constructing an angle of specified size. (When the teacher shows the class how to use a protractor, not only must the use be demonstrated but a verbal explanation of what is being done should accompany the demonstration.)

   Estimation of the size of an angle: In all problems it is desirable to estimate the answer in order to recognize a ridiculous answer obtained by calculation. A student may attempt to draw an angle of 60 degrees with his protractor. He should be able to look at the angle and know whether there is a major mistake in the size of the angle he has drawn. The exercise on pages 17 and 18 gives him practice in the work of estimating.

Constructions: The following are required for this course.

Copying an angle; bisecting an angle; bisecting a straight line; constructing a line perpendicular to another line from a point inside the line; and constructing a line through a given point parallel to a given line.

The student should not only be able to carry out these constructions but also describe how they are done. With large classes it is a useful exercise for one student to tell what to do while another at the blackboard carries out the instructions.

Triangles:

1. Definition of triangle: Note that a triangle is one kind of polygon.

2. Classification of triangles according to the number of equal sides: Scalene, isosceles, equilateral.

3. Naming the parts of a triangle: Some practice is necessary in naming the angles, sides and vertices of a triangle. The pupils become aware of the existence of the parts of the triangle.

4. Construction of triangles to certain specifications. (Pages 45, 48 and 53 of the text.) These exercises not only give practice in the drawing of accurate figures but may be used to show:

   (a) If two triangles have three sides of one equal to three sides of the other, the triangles are congruent.

   (b) If two triangles have two sides and the included angle of the one equal to two sides and the included angle of the other, the triangles are congruent.

   (c) If two triangles have two angles and a side of one equal to two angles and a corresponding side of the other, the two triangles are congruent.

   N.B. In this work it would seem advisable to make a freehand sketch showing the length of the known sides, the sizes of the known angles and the names of the vertices, before the accurate drawing is attempted.

The ideas associated with perpendicular lines may be brought to the fore in order to make the student more conscious of the following:

(a) There are two equal adjacent angles.

(b) There is a common arm.

(c) There is a common vertex.

Outdoor Work

The theodolite; the plumb level; construct a simple theodolite (page 49) and use it on problems such as appear on pages 51 and 52.

Length, Area and Volume (*Chapter II*)

A. Formulas for perimeters of rectangle, square, triangle, and circle; areas of rectangle, square, parallelogram, triangle, circle, ends of a cylinder and the curved surface; volume of a rectangular solid, volume of a cylinder.

B. Solution of problems by means of formula:

   1. The students should understand how to derive the formula by using geometric diagrams. N.B.—The textbook in many places attempts to have the students with a little help derive the formula for themselves.

   2. Repetition will be needed for the memorizing of the formula. The students should be taught to obtain the formula from a diagram any time they do not know it instantaneously. In all oral work it might

be best to have the students give the formula in words rather than in letters.

e.g. $V = lwh$ would be read "Volume of a rectangle solid equals the product of the length, width and height."

3. Suggested method for writing out the solution to the problem. In even the simplest problems the aim is not so much to obtain the correct answer (although this is important) as to have the pupils learn a method of attack.

Steps in the Solution:

(a) Write down the names of the quantities in the problem, one quantity to a line, with the value of the quantity following the name.

(b) Write down the symbols and their values. (The values are pure numbers.)

(c) Write down the formula.

(d) Substitute in the formula.

(e) Solve.

(f) Write out the answer in statement form.

(g) Check the problem.

Example: Problem—Find the area of a circle whose radius is seven inches.

Step (a) Radius is 7 in.
Step (b) $r = 7$
Step (c) $A = \pi r^2$
Step (d) $A = 22/7 \times 7 \times 7$
Step (e) $A = 154$
Step (f) Area of the circle is 154 sq. in.

In all problems the first task is to identify the quantities in the problem. It has been said that interpreting the problem is one of the biggest difficulties encountered by pupils. It is rather useless to ask the pupil to read the problem, for many let their eye go over the words without realizing what they are seeking. The suggestion here is to give them a specific aim for their reading, the finding of the names of the quantities in the problem. This may be easy in simple problems, but we are not concerned with the simplicity of the proceedings; we are concerned with the fact that it gets the pupil started on the solution and is fixing a good habit which will be invaluable in later more complicated problems.

It might be added that in mathematics one works with relationships between quantities and obviously this cannot be done until the quantities are identified. In later problems in which formulas are not used the pupil must look for relationships between quantities and these are sought by observing the names and meaning of the quantities rather than the values of the quantities.

C. Other Topics

1. Estimation of lengths and areas.

   The units of length and area, such as square yard, square foot, etc., should not be merely names to the pupils. They must have some idea of the size of these. This may be acquired by actually measuring the lengths or areas and by estimating lengths and areas.

2. Accuracy of measurement, rounding off numbers, number of digits in the product of approximate numbers.

   Pupils must realize that, no matter how carefully measurements are made, no matter how precise the instruments, all measurements are approximations for the true values. They must understand not only that 16.4 inches is an approximate answer, but also that in writing 16.4 inches as an answer you are saying that the true value lies between two values. In this case (16.4) the true value is greater than or equal to 16.35 in. and less than 16.45 in. This fact can be seen by realizing that the 16.4 is obtained by rounding off one of the following numbers: 16.44, 16.43, 16.42, 16.41, 16.40, 16.39, 16.38, 16.37, 16.36, 16.35.

   When the product of two or more approximate numbers is found, the product must be an approximation. Page 86 of the text deals with the number of significant figures that are in the product.

3. The Metric System: Pupils must have perceptions of the sizes of the units of length, area and volume, and see the relations between the units. A word such as centimeter must not be just a word. The pupil should be able to show approximately the length involved. These things can only be accomplished by using actual physical objects, such as a cube of wood one decimeter in length.

The Beginning of Algebra: (*Chapter III*)

1. The letters stand for numbers.

2. A formula such as $V = lwh$ which contains four letters, can be used to do four types of problems:

   Find V, given l, w and h,

   Find l, given V, w and h,

   Find w, given V, l and h,

   Find h, given V, l and w.

Not only this but all problems are done by using the same method, the method outlined above under the heading "Solution of problems by means of formula."

This can be illustrated by actually carrying through the solution of a number of problems. All the steps can again be reviewed and emphasis may be laid on the fact that the letters stand for numbers.

The teacher will have to give some work here on the solution of equations and may at this point provide motivation for the work of Chapter IV, "The Equation and Its Uses."

3. Literal numbers, terms, monomial, binomial, factors, numerical factor, literal factor, coefficient, like terms. The aim in learning these names is not to give formal definitions which the pupils may memorize without real understanding. The teacher should try to have the pupil grasp the idea (by use of examples and then attach the name to the idea). Literal and numerical factors and coefficients occur where there are products while terms, monomials and binomials occur where there are sums or differences. The pupil's mind must be made to look at the algebraic expression and realize whether it comprises a single term or several terms. This work is necessary for future progress in:
   (a) Division of a binomial by a monomial. (Consider the cartoon on page 313.)
   (b) Work in cancellation where the pupil must realize that there have to be factors and not terms in both the numerator and the denominator.
   (c) Multiplying and dividing both sides of an equation by the same number, say 5. Here every term, not every factor, on both sides of the equation is multiplied by 5. (In all examples constantly refer to meaning as suggested in the text, page 119.)

The Equation and Its Uses (Chapter IV)

Solution of equations. The objective is to get the unknown number by itself as one side of the equation. The pupil must constantly examine the side of the equation containing the unknown in order to decide what to do.

Avoid cross multiplication and transposing to avoid having pupils invent their own fantastic methods of operation. Transposing may come at a later stage when the pupil fully realizes what he is doing. A conscientious study of Chapter IV by the teacher, eager to help his students toward a position where they can solve problems under their own steam, will pay dividends. The text's suggestions in regard to problem solving are excellent.

Another suggested plan of attack (similar to the one in the text):

This plan parallels the one suggested for the work on the solution of problems by formula.

Step "a"—Write down the names of the quantities in the problem. The pupils will require help in identifying the quantities. They must come to realize what quantities are; that "apple" is not a quantity, but there are many quantities associated with apple—the diameter, the radius, the volume, the surface area, the weight, the value, etc.

In problems on coins these quantities occur: The number of nickels, the number of dimes, the number of quarters, the number of coins, the total value of the nickels, the total value of the dimes, the total value of the quarters, the total value of the coins. These quantities may more easily be recorded in a table:

	Number	Value in cents
Nickels		
Dimes		
Quarters		
Mixed coins		

In problems involving moving bodies the quantities may be recorded as in the following example:

	Distance in miles	Time in hrs.	Speed in m.p.h.
Slow train			
Fast train			

Here, of course, it is understood that the distance is the distance gone in the specified time.

In problems involving mixtures the quantities may be recorded so:

	No. of lb.	Value of 1 lb.	Total value
Cheap tea			
Expensive tea			
Mixed tea			

Step "b"—Representing the unknown quantities: Represent the unknown quantity by x inches or x hours or x lb., etc., and record opposite the names of the appropriate quantities the values that are given in the problem.

Step "c"—Filling in the values of the quantities: There still remain some values to fill in. These are obtained by using various relationships between the quantities that will be stated or implied in the problem.

In the above table regarding tea there is an obvious relationship across on each row (No. of lb. times value of 1 lb. equals the total value), and there are obvious relationships in columns (1) and (3) (No. of lb. of cheap tea plus No. of lb. of expensive tea equals the No. of lb. of mixed tea). (Total value of cheap tea plus total value of expensive tea equals value of mixture.)

A discussion along these lines without reference to any particular problem is invaluable to the students in helping them to identify the quantities and the relationships between them.

Step "d"—The equation: When the values of the quantities are obtained there always remains a relationship which can be used to obtain an equation. (Here the possibility of various solutions becomes obvious, for not every individual will select the same relationships left to use in obtaining an equation.) It is suggested that the relationship be written out in words and then the pupil will substitute the measures for the names of the quantities, thus:

Distance the slow train travels + distance the fast train travels = distance between the cities.

$$40x + 30x = 210$$

Step "e"—Solve the equation: The last line here would be $x = 3$, not $x = 3$ hr.

Step "f"—The answer: Write out the answer to the problem in statement form. Underline the answer.

Step "g"—Verify: By seeing that the answer satisfies all the conditions of the problem.

1. The nature of a formula
   (a) A symbolic statement of a rule.
   (b) Applies to any rectangle or any cylinder, or any circle.
   (c) Is a relation between the measures of two or more quantities. The pupil must come to realize: That if a formula contains four measures, and if three of them are known, the fourth can be obtained by substitution; that if there are four measures, four different types of problems can be solved and all of them by the same method of substitution.
   (d) Each specific formula is devised for use with certain units. See page 174.

2. The formula may be obtained in three ways:
   (a) Translating a rule from English into algebraic symbols.
   (b) By using tabulated data.
   (c) By using a geometric figure.

3. Obtaining formula from tabulated data:

A scientist may suspect that a relationship exists between two quantities. He measures corresponding values of the two quantities and places his data in a table. He is now faced with the problem set forth in the text, that of obtaining a formula from the tabulated data. This work in the text is easy if attacked in a systematic manner.

There are six cases in the work in this course:

(a) Where y is divided by x to give a constant value (or x is divided by y to give a constant value).

e.g.

x	1	2	3	4	5	6
y	18	36	54	72	90	108
y/x	18	18	18	18	18	18

$$y/x = 18$$
$$y = 18x$$

(b) Where y is multiplied by x to give a constant value:

e.g.

y	2	3	4	5	6
x	60	40	30	24	20
yx	120	120	120	120	120

$$yx = 120$$
$$y = 120/x$$

(c) Where y minus x (or x minus y) gives a constant value:

e.g.

y	9	10	11	12	13
x	1	2	3	4	5
$y - x$	8	8	8	8	8

$$y - x = 8$$
$$y = x + 8$$

(d) Where y added to x gives a constant value:

e.g.							
	y	19	18	17	16	15	14
	x	1	2	3	4	5	6
	$y + x$	20	20	20	20	20	20

$$y + x = 20$$
$$y = 20 - x$$

(e) Where y divided by x (or x divided by y) gives a quotient which is approximately constant. (It is constant within the limits of experimental error.)

e.g.							
	x	1	2	3	4	5	6
	y	2.5	5.1	7.3	10.1	12.7	15.2
	x/y	2.5	2.55	2.43	2.53	2.54	2.53

Here in order to reduce the rate of error the ratio associated with the greatest (and carefully measured) measures is used to obtain the formula:

$$y/x = 2.53$$
$$y = 2.53x$$

(f) Where y minus x gives a series of values, where the intervals between the values of the x's are equal. See page 197 No. 16.

e.g.							
	y	6	9	12	15	18	21
	x	1	2	3	4	5	6
	$y - x$	5	7	9	11	13	15

Intervals between x's are equal.
A series with a common difference of 2.

In this case try doubling the values of x before subtracting and see if you get a constant answer. If you do not obtain a constant value, try tripling the values of x and subtracting and see if you get a constant answer, etc., until a constant value is obtained. In this example:

y minus three times the value of x gives the constant answer of 3.

$$y - 3x = 3$$
$$y = 3x + 3$$

In attacking a problem where a formula is to be derived from a table try each procedure a, b, c, d, e, f, in turn until the formula is obtained. The pupil must realize it is not always possible to obtain a formula from a table of values.

4. Re-arranging formulas:

This section gives practice in solving for the unknown letter before substituting in the formula. Generally in the work of this course the pupils will find it easier to stay with the method of substituting and solving rather than solving for the unknown and then substituting. Where the solution of a number of similar problems is required as in:

Find r for each of the following:

$A = r^2$

A	14	25	30	70
r				

Then it is time saving to solve for r in the formula and then substitute in the resulting form.

5. Interpreting Formulas:

Show the effects produced by first, second and third powers as well as the inverse effect produced by a first power in the denominator. In addition to the suggestions in the textbook the method of substituting in a formula might be used to show these effects.

e.g. "Show that tripling the side of a square increased its area nine times." Reason this way: "To compare the areas it is necessary to find the areas,

so find them by substituting in the formula $A = S^2$

Area of original square $= A_1$
Side of original square $= S_1$

$A = A_1$
$S = S_1$

Formula: $A = S^2$
Substitute and get $A_1 = S_1^2$

Area of original square is $\underline{\underline{S_1^2}}$

Area of new square $= A_2$
Side of new square $= S_2$ or $3\ S^1$

$A = A_2$
$S = 3S_1$

Formula: $A = S^2$
Substitute and get: $A_2 = (3S_1)^2$

$A_2 = 9S_1^2$

Area of new square is $9S_1^2$

Area of new square $= \dfrac{9S_1^2}{S_1^2}, = 9$

When the effects produced by first, second and third powers and the inverse effect produced by a first power in the denominator are known, then the pupil will use the knowledge to do the problems on page 192. The pupil, as soon as he sees the formula $y = 6x^3$, should be able to tell what happens to the value of y when x is quadrupled.

Integers (Chapter VI) and Fractions (Chapter VII)

The material in the text is straightforward and needs no elaboration.

Signed Numbers (Chapter VIII)

Addition, subtraction, multiplication and division of signed numbers. The rules on page 271 of the text give a useful summary but are not to be memorized word by word by the pupil. The pupil must understand what is to be done and then state the procedure in his own words.

Graphs (Chapter IX)

Note the advice given on pages 277 and 278 regarding making graphs.

The following steps are suggested in making a graph from a formula: (The graph should be as large as the sheet of graph paper will allow in order to cut down the percentage error.)

1. Draw the axes.
2. Label the axes.

3. Place a heading on the graph, which in conjunction with the labels on the axes will show what the graph indicates.

4. Indicate on the axes the limits to which the axes scales have to go.

5. Decide on the scales for the axes and indicate them on your sketch. This step may be accomplished by marking off intervals on the axes which are to represent the larger intervals (of 10 small intervals) found on the graph paper and placing the appropriate numbers at ends of the intervals.

   The pupils will need some guidance in how to determine what scales to use:

   (a) The scales which allow one large interval on the graph paper to represent: 1, 2, 5, 10, and all multiples of 10 up to and including 100, 200, and 500, etc., are the easiest to read and so should be used. Convince the pupils of this by trying these scales and the ones omitted, 3, 4, 6, 7, 8, 9. There are some scales not included in 1, 2, 5, etc., which are quite easy to read; these may also be used.

   (b) The pupil will know how many large intervals can be used on a sheet of graph paper and still leave a good margin.

   (c) When the number of intervals available and the limit of the scale are known, then the first of the scales suggested in (a), which will meet these conditions, should be used.

   e.g. If the scale limits are 0 to 180 and there are 6 intervals, the scale which used 50 units per interval would be used. 6×50 brings you to 300 which is over 180, but the 20 scale would not do, for 6×20 only brings you to 120.

6. Make a table of values and plot the points.

   A straight line graph will be obtained if the formula is of the first degree in the variables. If one of the variables varies directly with the other you will obtain a straight line graph. These two ideas should be made clear to the pupils so that they may know when they are going to obtain a straight line graph. When it is known that a straight line graph is under consideration, only three points are required. (Two to obtain the line and one to check in the accuracy of the work.) These points should be far apart in order to secure accuracy.

More Complex Operations (Chapter X)

Substitution of signed numbers.

It is suggested that parentheses be placed around the negative number before it is substituted in the expression or equation; that the student be taught to do the substitution before any simplification is undertaken.

Some Everyday Uses of Arithmetic (Chapter XI)

This section is largely a review but the students must obtain a clear understanding of the situation and the quantities involved.

Geometry (*Chapter XII*)

It is not the function of this course to teach the nature of proof. Propositions given here are not to be proven; rather the students must have an understanding of the hypotheses and conclusions of the various statements.

In converse propositions (meaning examples, whether true or false), the pupils will find it easier to state the converse if they first state the original proposition in the "if . . . then" form.

Similar figures:

(a) Meaning of similar figures:

Two figures of the same shape but not necessarily of the same size are said to be similar.

(b) If two figures are similar then:

(1) the angles of the one are respectively equal to the angles of the other.

(2) The ratios of the corresponding sides are equal.

(c) If in two figures:

(1) The angles of the one are respectively equal to the angles of the other;

and

(2) The ratios of the corresponding sides are equal, then the two figures are similar.

N.B. Only one of these two conditions has to be known before you can conclude triangles are similar.

Vocabulary List of Significant Mathematical Terms.

(This list is arranged in the order in which the terms appear in the text book. This list will supplement the outline.)

CHAPTER I

†scale
line
dividers
†angle
degree
right angle
arms of an angle
†straight angle
‡†perpendicular
‡†bisect
plans
†centre
†radius
†diameter
†circle

CHAPTER II

†perimeter
circumference
pi.
periphery
composite numbers
prime factors
multiples
powers
†squares
‡perfect squares
square root
cube root
unit
†measure
quantity
rectangular solid
cube
†altitude
†base
nets of solids
significant digits
sector
planimeter
†formula

CHAPTER III

literal number
terms
monomial
binomial
polynomial
expression
numerical factor
literal factor
coefficient
like terms
‡parentheses
terms of a fraction
†numerator
†denominator

CHAPTER IV

equation
root equation

CHAPTER V

‡†formula
(See Ch. II)
subject of a formula
direct variation
indirect variation

CHAPTER VI

whole number
integer
digits
†place value
factors
odd numbers
even numbers
prime numbers

CHAPTER VII

fraction
†denominator
(see Ch. III)
†numerator
(see Ch. III)
terms of fraction
(see Ch. III)
proper
†improper
mixed number
reciprocal
complex fractions
†decimals
recurring decimals
terminating decimals
period of decimals

CHAPTER VIII

signed numbers
directed numbers
negative numbers
positive numbers
numerical value
absolute value

CHAPTER IX

pictographs
bar graphs
rectangular graph
circle graph
axes
origin

CHAPTER X

(no new terms)

CHAPTER XI

‡percent
rate
†base
percentage
cost price
‡margin
‡profit
overhead
operating expenses
‡†discount
list price
complementary
angle
‡congruent triangles
converse
‡parallel lines
vertically opposite
angles
adjacent angles
transversal
alternative angles
exterior angles
interior angles
corresponding
angles
‡ratio
similarity
pantograph
‡hypotenuse
†interest
†principal
‡amount
‡taxes
‡mills
insurance
‡face
risk
‡premium
†averages
frequency
distribution
mode
arithmetical mean
acute angle
obtuse angle
supplementary angle

‡Words and phrases which occur in the list for Grade VIII.

†Words and phrases which occur in the list for Grade VII.

Curric Guide

Date Due
